POSSIBILITIES FOR TRANSPORT ROUTES OPTIMIZATION IN SUPPLY CHAIN

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Introduction

Modern economy functions in a highly dynamic environment with limited resources potential. This explains the intention of the rational participants in the economy to search for opportunities for achieving competitive positions through effective use of the limited resources. One of the ways this goal to be achieved and economic growth to be assured is applying the “Supply Chain Management” conception. The essence of this conception is the cooperation and interrelationship between all participants involved in materials flow from the raw materials source to the consumer, that results in better economic returns.

The modern concept of supply chain management emphasizes on intercompany planning and optimization of resources when the relationship between the main company and other participants are determined. Nevertheless supply chain management is regarded as coordination of production, stocks, location and transportation among the participants in the supply chain. This aims to achieve the best gross results for the particular market.

The focus of this paper is on transportation that ensures the actual movement of material flows between the particular participants in the supply chain and as a result, transport costs form a relatively large share of the total logistics costs.

Transportation is considered as one of the essential functions in the logistics and “its basic economic functions are mainly related to creating optimal terms for normal reproduction process, for overcoming the distance between production and consumption while observing the economic principles or reducing costs and stocks storage time”.

The serious goals set to the supply chain management, and the fact that are observed economic principles for reducing costs and stocks storage time, that necessitates effective and scientifically based methods and approaches to be used. Such methods are, for example, economic and mathematical modeling of economic processes and events, which are characterized by their proven potential to ensure optimal results.

With this paper the author aims to make an attempt to suggest an economical and mathematical transport-type model and a method for its solving, which ensures

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the delivery of all loads from the departure place (supply point) to the delivery point (demand point) in the shortest possible time, while the transportation costs are within certain limits.

In order to achieve the paper’s objective, the following tasks need to be resolved:

- Constructing of economical and mathematical model to identify the optimal routes for loads transport from the departure place to the delivery point (destination point) in the supply chain, in the shortest possible time while the total transportation costs are limited.
- Theoretical argumentation of a method for defining an optimal solution to the thus constructed model.
- Demonstrating the effect of applying the suggested economical-mathematical model by using some sample data.

1. Formulation of the economic-mathematical model

Regarding the tough activities related to determination of routes for transporting of material flows in the supply chain, in order to achieve the best economic results in various areas, we can point out that in the specialized literature there are well developed theoretical and practical economical mathematical methods.

The most famous is the transportation problem model, whose purpose is to identify the optimal plan of transport when the departure points are defined with certain quantities of stocks available and delivery points are known together with their needs.

Usually in the transportation problems the optimality criterion is minimizing the total transport costs, i.e. when total transport cost from all departing points to all delivery points are known, it is necessary to determine such a plan of shipment, which satisfies the declared demands of the delivery points with the stocks quantities available in the departure points with minimum total transport costs. This model is known as transportation problem with value as a criterion. There are other well known modifications of the transportation problem with value as a criterion, which take into consideration certain peculiarities of the transport activities. For example, when it is impossible to accomplish direct shipment between particular departing point and a particular destination we proceed to so-called transportation problem with blocked shipments. Another example of modification of the transportation problem with value as a criterion is the two-stage transportation problem. We proceed to that type of problem in those cases when it is necessary the transported commodities to pass through intermediate points while being moved from the departing place to the destination point.

In some cases the specifics of the shipped loads and their processing could require the shortest possible time for distribution of goods between departure and destination points. In these cases, except the quantities available in the departure points and the quantities required in the delivery point, it is necessary to know the time needed for movement of goods from departure to the delivery places. Then the optimality criterion “minimum total transport cost” is being replaced with optimality criterion “minimum carriage time” and thus obtained model is called “transport task with time criterion”
The time for executing the whole plan for allocation of goods is considered to be the longest time for moving goods from a starting point to a delivery point, where there must be a shipment. To find optimal solutions to such problems are used well known methods of linear optimization.\(^3\)

Solving a transportation problem when calculating only one factor – such as transport costs or carriage time is not always reasonable. It is of interest to solve that problem considering both transport cost and time for implementation of the plan.

This is how we reached the following formulation of the problem.

In a supply chain it is necessary to transport some homogenous goods between two consecutive participants in the chain, when minimum timings are aimed and the total transport costs do not exceed certain value.

Let us accept we have \( m \) starting points (place of departure) \( A_1, A_2, \ldots, A_m \), with availability of stocks accordingly \( a_1, a_2, \ldots, a_m \) and \( n \) delivery points \( B_1, B_2, \ldots, B_n \) with declared necessities of that particular good \( b_1, b_2, \ldots, b_n \). The availability of stock is equal to volume of necessity of the goods, which need to be shipped between starting and delivery points:

\[
\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j,
\]

We presume that from every starting point it is possible to transport a load to every delivery point. Timings \( t_{ij} \) needed to transport arbitrary quantity of the good from point \( A_i \) \((i = 1 \ldots m)\) to point of delivery \( B_j \) \((j = 1 \ldots n)\) are known from the matrix:

\[
T = \begin{bmatrix}
    t_{11} & t_{12} & \cdots & t_{1m} \\
    t_{21} & t_{22} & \cdots & t_{2m} \\
    \vdots & \vdots & \ddots & \vdots \\
    t_{m1} & t_{m2} & \cdots & t_{mn}
\end{bmatrix}.
\]

We assume that these timings do not depend on the quantity of the load shipped, i.e. there are always enough vehicles available, so they can secure transportation of any volume of the goods.

With \( x_{ij} \) we denote the unknown quantity of a product which has to be transported form departure point \( i \) to delivery point \( j \) and we call the matrix \( X = [x_{ij}] \) a transport plan (Shipments plan).

Since all shipments in a plan are going to be finished at the moment when the longest shipment is completed, the time \( T \) will get the maximum value among all timings \( t_{ij} \) corresponding to filled in cells, i.e. to the positive \( x_{ij} \).

\[
T = \max_{i,j \mid x_{ij} \neq 0} t_{ij},
\]

where \( x_{ij} > 0 \) shows that we take not the maximum among all \( t_{ij} \), but only among those, whose corresponding quantities are different than zero.

With \( d_{ij} \) we denote those expenses, referring to delivery of a single unit of the load from starting point \( i \) to delivery point \( j \) and are described in the matrix

\[
D = [d_{ij}] = \begin{bmatrix}
d_{11} & d_{12} & \cdots & d_{1n} \\
d_{21} & d_{22} & \cdots & d_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
d_{m1} & d_{m2} & \cdots & d_{mn}
\end{bmatrix}.
\]

With \( d \) we denote all funds, intended for carrying out all the shipments in the supply chain.

Our task is to identify such a shipment plan, in which the whole demand of all delivery points is fully satisfied, all cargo is transported, the time of shipment is minimum and total transport costs do not exceed certain value.

When we formulated the problem in such a way, we will suggest the following economic-mathematical model

It must be found

\[
\min_{x_{ij} \geq 0} \max_{t_{ij}}
\]

under these conditions

\[
\sum_{j=1}^{n} x_{ij} = a_{i}, \, (i = 1 \div m),
\]

\[
\sum_{i=1}^{m} x_{ij} = b_{j}, \, (j = 1 \div n),
\]

\[
x_{ij} \geq 0, \, (i = 1 \div m, \, j = 1 \div n),
\]

with the additional limitation

\[
\sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} x_{ij} \leq d.
\]

It should be explicitly noted that \( d_{ij} \) and \( d \) could be given different economical interpretations, which makes the model (1) – (5) universal to a great extent.

The model (1) – (4) is widely known in the specialized literature as transport task with “time” criterion and the well-known methods could be used for solving it.\(^4\)

When we take into consideration the peculiarities in the structure of the economical-mathematical model (1) – (5), we suggest a special approach to its solving, which includes solving of a limited number of transportation problems with “value” as

a criterion. One of the advantages of the selected approach consists in the fact that in every further step in the procedure of calculations we can use information received from the previous steps. Another positive aspect of the suggested method that should be mentioned is the possibility to use variety of software products for solving a classic transportation task with “value” as a criterion.

It should be noted here, that the model should be perceived as a multiplicity of parameters and an aggregate of relations, when any of them determines the correlation between the meaning of a random subset and the parameters of the model. A model could be perfect in respect to the mathematical formalization, but if a method for its solving does not exist, then the model itself doesn’t have any practical value.

The method should be considered as a variety of parameters (variables), a set of operations above these parameters, a specifically defined, possible partial and/or implicit sequence of operations performed for a given array.

In that context, the method should be accepted as a process organization, based on specified set of operations, distinguishing the incoming parameters (a priori determined) and outgoing parameters (resultant). The process organization also determines the obtained results, based on the known values of the incoming parameters. At the same time the model represents a formal description of objects (the subject, occurrence, the event, the process). From the above said it becomes clear that the method answers the question “How”, and the model answers the question “What”.

2. A Method for finding an optimal solution of the suggested economical – mathematical model

Relying on the foregoing, we will give appropriate place to presenting an idea for a theoretical argumentation of proper method to solve the model (1) – (5). For that purpose, in the beginning a transportation problem (1) – (4) with time criterion must be solved. Let us assume that \( X^*_0 = \left[ x^*_0 \right] \) is an optimal solution of the (1) – (4) transportation problem. If plan \( X^*_0 \) satisfies the condition (5), then \( X^*_0 \) is an optimal solution to the transport task (1) – (5) as well.

Otherwise we denote with \( t_1, t_2, ..., t_s \) the different meanings of the parameters \( t_\gamma \) from the interval \( [t_1, t_2] \), where \( t_1 = \max_{\gamma} t_\gamma \) and \( t_s = \max_{\gamma} t_\gamma \). We form the multitudes

\[
U_s = \{i, j\} : t_\gamma \leq t_s, (s = 1 + k),
\]

and the matrices

\[
C^{(s)} = \left[ c^{(s)}_{\gamma \nu} \right], (s = 1 + k),
\]

where

\[
c^{(s)}_{\gamma \nu} = \begin{cases} 
 d_{\gamma \nu}, & \text{ako } (i, j) \in U_s^*; \\
 M, & \text{ako } (i, j) \notin U_s^*.
\end{cases}
\]

a sufficiently large positive number.
to each one of the multitudes \( U_s \) \((s = 1 + k)\), we are going to juxtapose one transportation problem from the type

\[
(L_s): \begin{align*}
\min Z(X) &= \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^{(s)} x_{ij}^s, \\
\sum_{j=1}^{n} x_{ij}^s &= a_i^s, \quad (i = 1 \div m), \\
\sum_{i=1}^{m} x_{ij}^s &= b_j^s, \quad (j = 1 \div n), \\
x_{ij}^s &\geq 0, \quad (i = 1 \div m, \quad j = 1 \div n).
\end{align*}
\]

At step one, we solve the problem \( L_1 \) based on the found plan \( X_0^* \). Let \( X_1^* = \|x_1^{(0)}\| \) is the optimal plan corresponding to \( L_1 \), and \( \{u_1^{(1)}, v_1^{(1)}\} \) – is the corresponding system of potentials.

1. If \( X_1^* \) meets the requirement (5), than \( X_1^* = \|x_1^{(0)}\| \) is the searched optimal solution.

Since, from proportion (7) due to the condition \( x_1^{(0)} > 0 \) it follows \((i, j) \in U_1\), therefore \( t_{x_1} = \max t_j \leq t_{x_1}^* \), i.e. \( X_1^* \) also appears to be a solution of the problem (1) – (4).

Therefore, \( X_1^* \) is an optimal plan for problem (1) – (5) as well.

2. Let’s assume that the found \( X_1^* \) does not satisfy the restrictive condition (5).

We determine the evaluations

\[ \alpha_{ij}^{(1)} = v_{ij}^{(1)} - u_{ij}^{(1)} - c_{ij}^{(1)} \quad \forall i = 1 \div m, \quad j = 1 \div n. \]

There are two possible cases for \( \alpha_{ij}^{(1)} \):

a) All \( \alpha_{ij}^{(1)} \leq 0 \). Then when considering (7) we make the conclusion that \( X_1^* \) is an optimal plan for the \( L_1 \) problem with coefficients in the targeted function \( c_{ij}^{(1)} = d_{ij} \).

Since \( X_1^* \) does not satisfy the condition (5), a plan for the problem (1)-(4) that satisfy (5) does not exist. Therefore the model (1) – (5) is unsolvable due to incompatibility of the conditions (2) – (5)

b) There is at least one evaluation \( \alpha_{ij}^{(1)} > 0 \). In this case transition to a new plan is possible.

3. Let us accept that on the step \( s \) we have found plan \( X_{s-1}^* \) (not satisfying condition (5)) and potentials \( \{u_i^{(s-1)}, v_j^{(s-1)}\} \), and at least one of the \( \alpha_{ij}^{(s-1)} = v_{ij}^{(s-1)} - u_{ij}^{(s-1)} - c_{ij} \) is positive. Then, when using the \( X_{s-1}^* \) plan and the system of potentials \( \{u_i^{(s-1)}, v_j^{(s-1)}\} \) we can move on to solving the problem \( L_s \).
Let’s analyze the optimal plan $X^*$ and the system of potentials $(u^{(s)}_i, v^{(s)}_j)$ of the transportation problem $L$. There are two possible cases:

a) If $X^*$ satisfies the condition (5), then $X^*$ is an optimal plan for the problem (1) – (5).

b) If $X^*$ does not satisfy the condition (5), then we have to calculate the system of potentials $(u^{(s)}_i, v^{(s)}_j)$, aiming to determine the

$$\alpha^{(s)}_{ij} = v^{(s)}_j - u^{(s)}_i - c_{ij} \quad i = 1 \div m, j = 1 \div n.$$  

If all $\alpha^{(s)}_{ij} \leq 0$, then the model (1) – (5) is unsolvable due to incapability of the restrictive conditions. In this case the procedure of calculation ends.

If there is at least one $\alpha^{(s)}_{ij} > 0$ we move on to the next step ($s+1$).

The basis of the method for solving the (1) – (5) model includes the following two characteristics:

1) Let $X^*$ is a solution of the transportation problem $L$ and if

$$\sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij} x_{ij} > d \text{ when } s = 1 \div k - 1;$$

$$\sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij} x_{ij} \leq d \text{ when } s = k, \text{ then }$$

$$X^* = \|x^{(k)}_{ij}\| \text{ is an optimal plan of the model (1) – (5).}$$

2) The procedure of calculation ends after a finite number of steps in the cases when either an optimal plan is found or insolubility of the (1) – (5) model is determined.

3. Using of economic-mathematical model with sample data

We will demonstrate the presented above model and the method for its solving using the following sample data.

There are three starting points $A_1$, $A_2$, and $A_3$, with stocks available in them respectively $a_1 = 100$ units, $a_2 = 70$ units, $a_3 = 80$ units and four accepting points $B_1$, $B_2$, $B_3$, and $B_4$ with declared the following needs: from $b_1 = 50$ units, $b_2 = 60$ units, $b_3 = 40$ units, $b_4 = 100$ units.

The total transport costs for all deliveries from departure points to destination points must not exceed the value of $d = 750$ units.

The times $t_{ij}$ for transporting a random stock quantity from starting point $i$ ($i = 1 \div 3$) to delivery point $j$ ($j = 1 \div 4$) are known and are determined with the matrix

$$T = \|t_{ij}\| = \begin{bmatrix} 2 & 5 & 6 & 1 \\ 3 & 4 & 8 & 3 \\ 5 & 6 & 2 & 4 \end{bmatrix}.$$  

The expenses $d_{ij}$, made for delivery of one unit of cargo from starting point $i$ to delivery point $j$ are given with the matrix

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* Shapiro, Dzh. Modelirovanie cepi postavok. Per. s angl. pod red. V.S. Lukinskogo. SPb.: Piter, 2006, s. 382.
Initially we solve the transportation problem (1) – (4) with “time” as a criterion. The optimal solution is presented in tabl. 1.

Table 1

<table>
<thead>
<tr>
<th>A_i</th>
<th>B_j</th>
<th>B_1</th>
<th>B_2</th>
<th>B_3</th>
<th>B_4</th>
<th>a_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>A_1</td>
<td>2</td>
<td>40</td>
<td>3</td>
<td>6</td>
<td>60</td>
<td>100</td>
</tr>
<tr>
<td>A_2</td>
<td>3</td>
<td>10</td>
<td>4</td>
<td>8</td>
<td>1</td>
<td>70</td>
</tr>
<tr>
<td>A_3</td>
<td>5</td>
<td>10</td>
<td>6</td>
<td>2</td>
<td>1</td>
<td>80</td>
</tr>
<tr>
<td>b_j</td>
<td>50</td>
<td>60</td>
<td>40</td>
<td>100</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The obtained shipment plan shows that from the first departure point 40 units of cargo must be transported to the first delivery point and 60 units of cargo must be transported to the fourth delivery point. From the second starting point 10 units of cargo must be transported to the first delivery point and 60 units to the second delivery point. From the third starting point 40 units must be transported to the third and the forth delivery points.

Additionally, the maximum time for the shipments is \( t_1 = \max t_{ij} = 4 \), and the shipment costs add up to

\[
\sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} x_{ij}^{(0)} = 40.3 + 60.0 + 10.3 + 60.5 + 40.3 + 40.6 = 810 \text{ units}.
\]

The total costs (810 units) exceed the limitation of 750 units, that means that this is not the optimal shipment plan we seek for.

We determine those shipment timings who exceed \( t_1 = \max t_{ij} = 4 \):

\( t_2 = 5 \) (in cells (1,2) and (3,1)),
\( t_3 = 6 \) (in cells (1,3) and (3,2)),
\( t_4 = 8 \) (in cell (2,3)).
We form the multitudes \( U_i \) with those cells from the table whose values of shipment time is not greater than \( t_i \):

\[
U_1 = \{(1,1),(1,4),(2,1),(2,2),(2,4),(3,3),(3,4)\},
\]

\[
U_2 = \{(1,1),(1,2),(1,4),(2,1),(2,2),(2,4),(3,1),(3,3),(3,4)\},
\]

\[
U_3 = U_2 \cup \{(1,3),(3,2)\},
\]

\[
U_4 = U_3 \cup \{(2,3)\}.
\]

As for the found plan

\[
X^*_o = \begin{bmatrix}
40 & 0 & 0 & 60 \\
10 & 60 & 0 & 0 \\
0 & 0 & 40 & 40
\end{bmatrix}
\]

condition (5) is not satisfied, we move to the next step.

Step I. We find the optimal plan for the transportation problem with time criterion (considering the transportation costs), when those costs \( d_{ij} \) for shipment from \( i \) to \( j \) for which the time is equal to 4 are replaced with \( M \) (fig. 2).

<table>
<thead>
<tr>
<th>( u_{ij}^{(1)} )</th>
<th>( v_{ij}^{(1)} = 3 )</th>
<th>( v_{ij}^{(1)} = 5 )</th>
<th>( v_{ij}^{(1)} = -3 )</th>
<th>( v_{ij}^{(1)} = 0 )</th>
<th>( a_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u_1^{(1)} = 0 )</td>
<td>40</td>
<td>( M )</td>
<td>( M )</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>( u_2^{(1)} = 0 )</td>
<td>10</td>
<td>3</td>
<td>5</td>
<td>( M )</td>
<td>4</td>
</tr>
<tr>
<td>( u_3^{(1)} = -6 )</td>
<td>( M )</td>
<td>( M )</td>
<td>( 40 )</td>
<td>40</td>
<td>6</td>
</tr>
</tbody>
</table>

We come to a shipment plan

\[
X^*_o = \begin{bmatrix}
40 & 0 & 0 & 60 \\
10 & 60 & 0 & 0 \\
0 & 0 & 40 & 40
\end{bmatrix}
\]

The obtained optimal plan does not satisfy the additional condition in the problem, therefore we proceed to the second step. We find the optimal plan of the transportation problem using “value” as a criterion. The costs \( d_{ij} \) for shipment from \( i \) to \( j \) for which the time is equal to 5 are replaced with \( M \).
The next table 3 presents the optimal shipment plan achieved in this step.

**Table 3**

**Optimal plan of the transportation problem at step II**

<table>
<thead>
<tr>
<th>$u_i$</th>
<th>$v_i=0$</th>
<th>$v_i=2$</th>
<th>$v_i=-3$</th>
<th>$v_i=0$</th>
<th>$a_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_1 = 0$</td>
<td>3</td>
<td>40</td>
<td>M</td>
<td>60</td>
<td>0</td>
</tr>
<tr>
<td>$u_2 = -3$</td>
<td>3</td>
<td>20</td>
<td>M</td>
<td>4</td>
<td>70</td>
</tr>
<tr>
<td>$u_3 = -6$</td>
<td>10</td>
<td>M</td>
<td>40</td>
<td>6</td>
<td>80</td>
</tr>
<tr>
<td>$b_j$</td>
<td>50</td>
<td>60</td>
<td>40</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

and is described by the matrix $X^*_2 = \begin{pmatrix} 0 & 40 & 0 & 60 \\ 50 & 20 & 0 & 0 \\ 0 & 0 & 40 & 40 \end{pmatrix}$.

The costs for implementing this shipment plan are

$$\sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij}x_{ij}^*(t) = 690$$ units.

This value satisfies the imposed additional condition in the problem – the shipment plan costs not to exceed 750 units. Therefore, $X^*_2$ is the searched optimal solution of the problem and it is implemented for time $t_{opt} = 5$.

According to the obtained optimal shipment plan from the first starting point 40 units of goods must be transported to the second delivery point and 60 units must be transported to the fourth delivery point. From the second starting point 50 units of goods must be transported to the first delivery point and 20 units must be transported to the second delivery point. From the third departure point 40 units should be sent to the third delivery point and 40 units – to the forth one. The time for implementing that shipment plan is $t_{opt} = 5$, and the costs will be 690 units. Thus obtained results show that the additionally imposed restriction ensures significant total cost reduction for implementing that particular shipment plan at the expense of only one unit time increase.

**Conclusion**

In this paper the author’s attention is focused on the possibilities to identify optimal routes for transportation of loads in the supply chains, calculating the shipment time and costs. An economic-mathematical model is constructed for identifying of optimal transport routes in cases when the time for transportation is not the main criterion for optimization. The author suggests a method for finding an optimal solution of the constructed model and using sample data demonstrates how its implementation may
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result in an increase of the economic effect of transportation activity in the supply chain.

The good knowledge of the economic-mathematical modelling allows adaptation of existing models and constructing new ones according to the specifics of a particular business activity, while applying them provides better economic results for the organizations.

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Abstracts

The attention of the author in the present work is directed at the possibilities for optimizing some of the main activities in supply chains, namely activities connected with the transportation of material flows between the participants in the chain. There is made an attempt at putting forward an economic and mathematical model of a transport type, through which there is ensured the delivery of all cargo from the points of departure to reception points in the supply chain in the shortest possible time, and the cost of transportation is within certain preset limits.

Keywords: transportation, optimum routes, criterion time.